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CALORIMETRIC HEATING-RATE PROBE FOR MAXIMUM-RESPONSE-TIME INTERVAL

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CALORIMETRIC HEATING-RATE PROBE FOR MAXIMUM-RESPONSE-TIME INTERVAL

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A desirable feature of calorimetric probes for very high heating-rate environments is the longest possible interval of linear temperature-time response. It is the purpose of this note to indicate a design for a calorimetric probe that will maximize the time interval of linear response.

A typical aerodynamic heating-rate probe is shown in Fig. 1. The frontal area of the calorimetric slug is small enough that only stagnation-point heat transfer is measured. Consider the probe to be a finite slab of material with a uniform heating rate at the front face and to be insulated on the sides and at the back face, thus reducing the problem to one-dimensional heat conduction as shown in Fig. 2.

The solution of the one-dimensional heat conduction equation

$$\frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha}\frac{\partial t}{\partial \theta} \tag{1}$$

subject to the boundary conditions

$x = 0$	$q = \text{const}$
$x = \delta$	$\partial t/\partial x = 0$
$\theta = 0$	$t = 0$

gives the temperature distribution in the model of Fig. 2. The solution is given by Carslaw and Jaeger.[1] The variation of temperature with time is plotted by Schneider,[2] as shown in Fig. 3.

From Fig. 3, and for a given material, a value of θ_1, the time required for the back face of the slug to reach a state of linear temperature increase with time, can be established. The time for the back face to undergo its initial transient may be taken from Fig. 3 as

$$\theta_1 = \frac{0.35\ \delta^2}{\alpha} \tag{2}$$

From the Carslaw and Jaeger solution, the front-face temperature after the initial transient is also a linear function of time, and is given as

$$t = \frac{\delta q}{k}\left(\frac{\alpha\theta}{\delta^2} + \frac{1}{3}\right) \tag{3}$$

The front face reaches a maximum allowable temperature t_m in time θ_2. Equation (3) then yields:

$$\theta_2 = \frac{\delta^2}{\alpha}\left(\frac{kt_m}{\delta q} - \frac{1}{3}\right)$$

In deducing heating rate from the temperature-time response curve, the following equation is employed:

$$q = \rho\delta c\ \frac{\Delta t}{\Delta\theta} \tag{4}$$

For extremely high heating rates, the time interval $\Delta\theta = \theta_2 - \theta_1$ (the linear portion of the temperature-time response of the attached thermocouple) can be very small, and, thus, difficult to measure. In optimizing a probe design, it is therefore desirable to have $\theta_2 - \theta_1$ a maximum.

Combining Eqs. (2) and (3) one has

$$\Delta\theta = \left(\frac{kt_m}{\alpha q}\right)\delta - \frac{0.683\ \delta^2}{\alpha} \tag{5}$$

The quantity $\Delta\theta$ may now be maximized as a function of the thickness δ:

$$\frac{d(\Delta\theta)}{d\delta} = \frac{kt_m}{\alpha q} - \frac{2(0.683)\delta}{\alpha} = 0$$

Therefore, the optimum value of thickness becomes

$$\delta_{optimum} = \frac{kt_m}{1.366\ q} \tag{6}$$

After substitution of Eq. (6) into Eq. (5), the maximum time interval is found to be:

$$\Delta\theta_{max} = 0.366\ \frac{k^2 t_m{}^2}{\alpha q^2} \tag{7}$$

Equation (5) may, for convenience, be normalized to yield

$$\frac{\Delta\theta}{\Delta\theta_{max}} = \frac{2\delta}{\delta_{optimum}} - \left(\frac{\delta}{\delta_{optimum}}\right)^2 \tag{8}$$

Equation (8) is plotted in Fig. 4.

The calorimetric probe designed by means of Eq. (6) will give the longest possible linear temperature-time response for a given material, allowable front-face temperature rise, and fixed heating rate.

NOMENCLATURE

c	slug specific heat
k	thermal conductivity
q	heating rate
t	temperature
t_m	maximum allowable front-face temperature
x	length dimension
α	thermal diffusivity

δ length of slug

ρ slug density

θ time

REFERENCES

1. Carslaw, H. S., and Jaeger, J. C.: Conduction of Heat in Solids. Second ed., 1959, Oxford at the Clarendon Press.

2. Schneider, P. J.: Temperature Response Charts. John Wiley and Sons, New York, 1963.

FIGURE LEGENDS

Fig. 1.- Aerodynamic heating-rate probe.

Fig. 2.- One-dimensional model.

Fig. 3.- Temperature response of a plate.

Fig. 4.- Plot of Eq. (7).

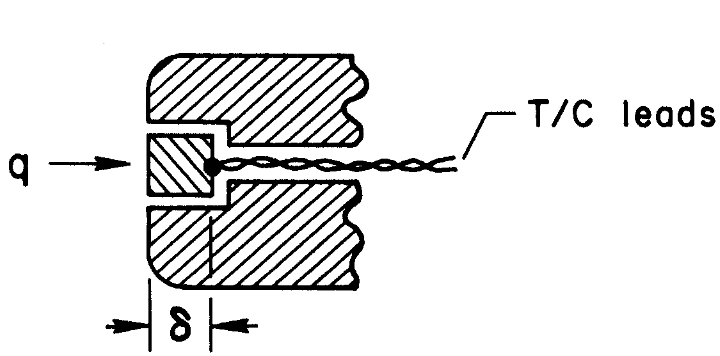

Fig. 1.- Aerodynamic heating rate probe.

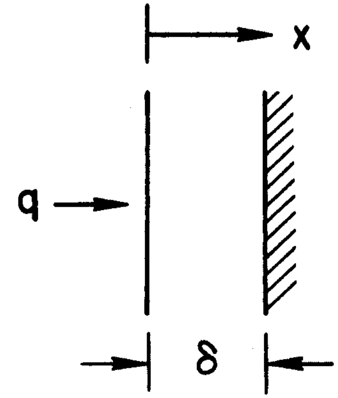

Fig. 2.- One-dimensional model.

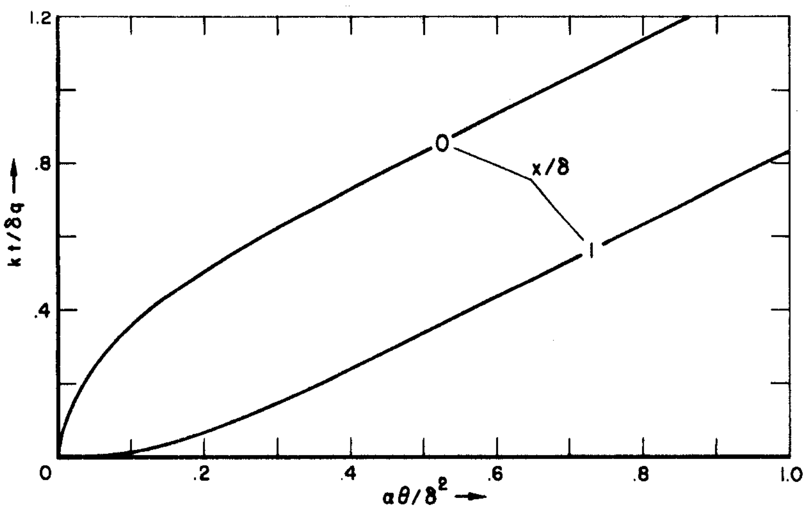

Fig. 3.- Temperature response of a plate.

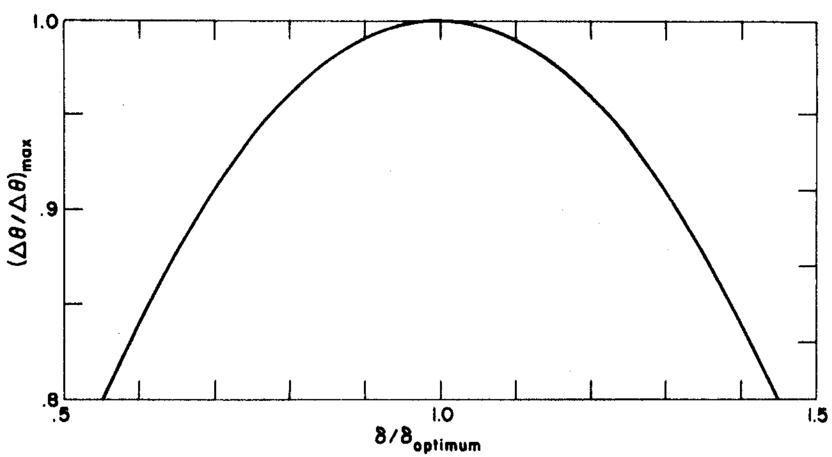

Fig. 4.- Plot of Eq. (7).

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